# S5 MATHEMATICS HOLIDAY PACKAGE TEST (100 MARKS)

**SECTION A: Answer all questions in this section.** [55marks]

1. The sum $(S_n)$ of the first $n$ terms of the arithmetic progression is [2marks]
   a. $\frac{n}{2}(u_n - u_1)$
   b. $\frac{n}{2}(u_1 + du_n)$
   c. $\frac{n}{2}(u_1 + u_n)$
   d. $\frac{n}{2}(2u_1 + u_n)$

2. Newton Raphson method for solving equation by numerical method is [2marks]
   a. $x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}$
   b. $x_{n-1} = x_n - \frac{f(x_n)}{f'(x_n)}$
   c. $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
   d. $x_{n-1} = x_n + \frac{f(x_n)}{f'(x_n)}$

3. The magnitude of $\vec{v} = (x, y, z)$ is [2marks]
   a. $\sqrt[3]{x^2 + y^2 + z^2}$
   b. $x^2 - y^2 + z^2$
   c. $\sqrt{x^2 + y^2 - z^2}$
   d. $\sqrt{x^2 + y^2 + z^2}$

4. Choose the arithmetic progressions from the options below: [4marks]
   a. 2, 4, 8, 10, 16, …
   b. 17, 14, 11, 8, …
   c. 2, 6, 10, 14, …
   d. -2, -4, 6, 8, …

5. Select two intervals in which the roots of $x^2 - 3x + 1 = 0$ are located: [4marks]
   **a.** ]0,1[
   **b.** ]3,4[
   **c.** ]1,2[
   **d.** ]2,3[

6. The tangent function $\tan(x)$ has
   a. Domain =? [2marks]
   b. Range =? [1mark]

7. Write **True** if the statement is correct or **False** otherwise:
   a. $f(x) = \frac{x + sinx}{x^2}$ is even function [2marks]
   b. $g(x) = \frac{1 - cos^3 x}{sin^2 x}$ is odd [2marks]

8. The value of $x$ if $\tan\left(x + \frac{\pi}{2}\right) = 1$ is: [3marks]
   a. $-\frac{\pi}{4} + \pi k$ , $k \in \mathbb{Z}$
   b. $\frac{\pi}{3} + \pi k$, $k \in \mathbb{Z}$

c.  $-\frac{\pi}{4} + 2\pi k, \ k \in \mathbb{Z}$

d.  $\frac{\pi}{6} + 2\pi k, \ k \in \mathbb{Z}$

9.  A. Select all trigonometric functions from the list below                    **[2marks]**

   i.   $y = sin2x$

   ii.  $y = \csc^{-1} 2x$

   iii. $y = \sec^{-1} 2x$

   iv.  $y = \tan x$

   **B.** Match the function with its corresponding derivative                    **[3marks]**

| Functions | Derivative |
|-----------|-----------|
| a.  $\cos^{-1} x$ | 1.  $\frac{1}{1+x^2}$ |
| b.  $\tan^{-1} x$ | 2.  $\frac{-1}{\sqrt{1-x^2}}$ |
| c.  $\sec^{-1} x$ | 3.  $\frac{1}{x\sqrt{x^2-1}}$ |

10. The best solution of the equation: $\log_2(x + 3) - \log_2 x = 1$ is        **[3marks]**

   a.  $x = 3$

   b.  $x > 3$

   c.  $x > 0$

   d.  $x < 3$

11. The 2 best solutions of $2sinx + \sqrt{3} = 0$ in the interval $[0, 2\pi]$ are   **[4marks]**

   a.  $\frac{\pi}{3} + 2\pi k$

   b.  $\frac{4\pi}{3} + 2\pi k$

   c.  $\frac{2\pi}{6} + 2\pi k$

   d.  $\frac{5\pi}{3} + 2\pi k$

12. Evaluate $\lim\limits_{x \to 0} \frac{sin^{-1}x^2}{(sin^{-1}x)^2}$          **[5marks]**

13. The function of $y = 3sinx + 2$ has

   a. The maximum value at $y$ =?                                **[2marks]**

   b. The minimum value at $y$ =?                                **[2marks]**

14. It is given that the initial population of a city $p_o$ is 10000 people; the exponential growth rate $(r)$ is 3%per year, $t$ is the time in years and $p_t$ is the population at time$t$.
   Create the population growth model of the given data and hence, estimate the population after 5 years.                                                       **[5marks]**

15. The 3rd term and 8th term of arithmetic progression are 5 and 15 respectively, find

   a.  The common difference                                     **[2marks]**

   b.  The 1st term                                              **[1marks]**

   c.  The sum of the first 20 terms                             **[2marks]**

**SECTION B: ATTEMPT ONLY THREE QUESTIONS OF YOUR CHOICE**                    **[45MARKS]**

16. A. By using determinant method Show that $\vec{u} = (1,2,3), \vec{v} = (0,1,2) \ and \ \vec{w} = (0,0,1)$ are linearly independent                                             **[5marks]**

   B. Find the volume of the tetrahedron whose vertices are $A(2,-1,-3), B(4,1,3), C(3,2,-1) and \ D(1,4,2)$
                                                                 **[10marks]**

17.

a. Use factorization, simplification and hospital rule to show that $\lim\limits_{x\to 0}\frac{x^2-sin^2x}{x^2-x\sin x}=2$ **[8marks]**

b. A circle of radius 3m is divided into many parts , one part is considered only and the angle $\theta$ between two radii is $15^0$.as shown below .Find the area of the figure below **[7marks]**
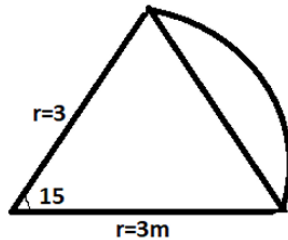


18.
   a. Evaluate the exact value of $\tan 75^0$ using trigonometric identity **[5marks]**

   b. solve the trigonometric equation for $0 \leq x \leq 2\pi$:

    $sin2x + cosx = 0$ **[5marks]**

   c. Find the derivative of $y = tan^{-1}\left(\frac{x-1}{x+1}\right)$ with respect to $x$ **[5marks]**

19.
   a. express $cos4x$ in terms of $cosx$ only **[6marks]**

   b. solve the inequality for $0 \leq x \leq 360$

   $cosx > \frac{1}{2}$ **[6marks]**

   c. Ferris wheel with a radius of 10metres at a constant speed, if a rider starts at the bottom and their height $h(t)$ (in meters) at time $t$ (in seconds) is modeled by :

   $h(t) = 10 - 10cos(\frac{\pi}{15}t)$

   Find the rider's height at $t = 10$ seconds. **[3marks]**

20.In KABEZA village , after her 9 observations about farming , UMULISA saw that in every house observed, where there are a number $x$ of cows there also $y$ domestic ducks, and then she got the following results :

| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Y | 4 | 8 | 4 | 12 | 10 | 14 | 16 | 6 | 18 |

a. Represent this information graphically in scatter diagram **[5marks]**

b. Find regression line equation $y$ $on$ $x$ and estimate the number of domestic ducks when the the number of cows is 12 **[6marks]**

c. Find correlation coefficient between the variation of the number of cows and the number of domestic ducks and comment on it. **[4marks]**

**GOOD LUCK!!!**